

# SPIN POLARIZABILITIES OF THE NUCLEON

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Spin polarizabilities of the nucleon ( $\gamma_i$ ) are discussed in the framework of fixed- $t$  and backward-angle dispersion relations, chiral perturbation theory, and the non-relativistic quark model. Calculations with the dispersion relations generally confirm findings from HBChPT and disagree with a recent experimental result from LEGS for the backward spin polarizability.

## 1 Introduction

With recent progress in developing and using effective field theories, a practical knowledge of various low-energy parameters of hadrons and their interactions becomes of high current interest. Among such parameters are the dipole electric and magnetic polarizabilities of the nucleon,  $\alpha_E$  and  $\beta_M$ , and the so-called spin (or vector) polarizabilities  $\gamma_{E1}$ ,  $\gamma_{M1}$ ,  $\gamma_{E2}$ ,  $\gamma_{M2}$ . They characterize a low-energy behavior of the nucleon Compton scattering amplitude up to order  $\mathcal{O}(\omega^3)$  and correspond to the following effective interaction of the nucleon's internal degrees of freedom with the probing soft electromagnetic field:<sup>1</sup>

$$\begin{aligned} \frac{1}{4\pi} H_{\text{eff}} = & -\frac{1}{2}\alpha_E \vec{E}^2 - \frac{1}{2}\beta_M \vec{H}^2 - \frac{1}{2}\gamma_{E1} \vec{\sigma} \cdot \vec{E} \times \frac{\partial \vec{E}}{\partial t} - \frac{1}{2}\gamma_{M1} \vec{\sigma} \cdot \vec{H} \times \frac{\partial \vec{H}}{\partial t} \\ & + \gamma_{E2} E_{ij} \sigma_i H_j - \gamma_{M2} H_{ij} \sigma_i E_j + \dots \end{aligned} \quad (1)$$

Here  $E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$  and  $H_{ij} = \frac{1}{2}(\nabla_i H_j + \nabla_j H_i)$  are the quadrupole strengths of the electric  $\vec{E}$  and magnetic  $\vec{H}$  fields, and the omitted terms are of higher order in the photon energy  $\omega$ . In particular, the low-energy  $\gamma N$  scattering amplitude at forward or backward scattering angles reads in terms of these polarizabilities:

$$f(0^\circ) = f_B(0^\circ) + \omega^2(\alpha_E + \beta_M) \vec{\epsilon}' \cdot \vec{\epsilon} + i\omega^3 \gamma \vec{\sigma} \cdot \vec{\epsilon}' \times \vec{\epsilon} + \dots, \quad (2)$$

$$f(180^\circ) = f_B(180^\circ) + \omega^2(\alpha_E - \beta_M) \vec{\epsilon}' \cdot \vec{\epsilon} + i\omega^3 \gamma_\pi \vec{\sigma} \cdot \vec{\epsilon}' \times \vec{\epsilon} + \dots, \quad (3)$$

where the Born term  $f_B = -(e^2 Z^2 / 4\pi M) \vec{\epsilon}' \cdot \vec{\epsilon} + \dots$  is determined by the electric charge  $eZ$ , the mass  $M$ , and the anomalous magnetic moment of the nucleon, and the quantities

$$\gamma = -\gamma_{E1} - \gamma_{M1} - \gamma_{E2} - \gamma_{M2}, \quad (4)$$

$$\gamma_\pi = -\gamma_{E1} + \gamma_{M1} + \gamma_{E2} - \gamma_{M2} \quad (5)$$

are the so-called forward and backward spin polarizabilities.

For the proton, the dipole polarizabilities  $\alpha_E$  and  $\beta_M$  are reasonably well determined by experiments on low-energy  $\gamma p$  scattering done in Moscow, Urbana-Champaign, Mainz, and Saskatoon (see <sup>2</sup> and references therein). All the spin polarizabilities can, in principle, be determined in the next-generation experiments on  $\gamma p$  scattering with the (circularly) polarized beam and target. Since fixed- $t$  dispersion relations predict <sup>1,3</sup> rather reliably three of the four spin polarizabilities (the exception is  $\gamma_\pi$ ), the available data on *unpolarized* proton Compton scattering were used to extract the remaining parameter  $\gamma_\pi$  giving <sup>4</sup>

$$\gamma_\pi = -27.1 \pm 3.4, \quad (6)$$

where the units hereafter are  $10^{-4} \text{ fm}^4$  and all errors, including systematic and model-dependent uncertainties, are summed up in quadrature. This finding challenges the existing theories (including HBChPT <sup>5</sup> and dispersion relations <sup>6</sup>), because theoretically a much larger  $\gamma_\pi \simeq -37$  to  $-40$  is anticipated. In the following, we briefly review and compare arguments of these theoretical frameworks and add another argument in their support which is based on the non-relativistic quark model.

## 2 Spin polarizabilities in HBChPT

To leading non-vanishing order  $\mathcal{O}(p^3)$ , the structure-dependent (non-Born) part of the Compton scattering amplitude is determined by diagrams of the effective chiral Lagrangian with one-pion loop and with the  $t$ -channel  $\pi^0$ -exchange. No free parameters, except for the standard set of them including the pion mass  $m_\pi$ , the pion decay constant  $f_\pi = 92.4 \text{ MeV}$ , and the nucleon axial coupling  $g_A = 1.26$ , appear to this order. Explicitly, the spin polarizabilities in the chiral limit of  $m_\pi \rightarrow 0$  are equal to <sup>7</sup>

$$\gamma_{E1} = -5X + X_a, \quad \gamma_{M1} = -X - X_a, \quad \gamma_{E2} = X - X_a, \quad \gamma_{M2} = X + X_a, \quad (7)$$

where  $X$  and  $X_a$  represent the loop and  $\pi^0$ -exchange contributions,  $\gamma_i^{(\pi N)}$  and  $\gamma_i^{(\pi^0)}$ , which are of second and first order in  $g_A$ , respectively:

$$X = \frac{e^2 g_A^2}{384 \pi^3 f_\pi^2 m_\pi^2} = 1.11 \times 10^{-4} \text{ fm}^4, \quad X_a = \frac{e^2 g_A \tau_3}{32 \pi^3 f_\pi^2 m_\pi^2} = \pm 11.3 \times 10^{-4} \text{ fm}^4 \quad (8)$$

( $X_a$  is positive for the proton,  $\tau_3 = +1$ ).

Actually, the leading-order approximation (7) represents properties of the static nucleon with a polarizable perturbative pion cloud. Quantitatively, such a simple picture does not work well and it needs further corrections, such as

the nucleon recoil and the  $\Delta$ -isobar excitation, which formally appear in higher orders of the chiral expansion. As quick means to include the phenomenologically important  $\Delta$  contribution, a modified expansion was proposed, in which the  $N\Delta$  mass splitting  $\Delta = M_\Delta - M \simeq 2m_\pi$  is counted as the value of order  $\mathcal{O}(p)$  too (or, a small energy scale  $\epsilon = \mathcal{O}(m_\pi, \Delta)$ ). Then, not only the  $\Delta$ -pole contribution, but also contributions  $\gamma_i^{(\pi\Delta)}$  of 1-loops with intermediate  $\pi\Delta$  states have to be kept to the same order. Numerically, however, the  $\pi\Delta$  loops contribute little to the spin polarizabilities,<sup>5</sup> so that a simplified result of such an approach is reduced to adding the  $\Delta$ -pole contribution to the dipole magnetic spin polarizability  $\gamma_{M1}$ :

$$\gamma_{M1}^{(\Delta)} = \frac{\mu_{N\Delta}^2}{4\pi\Delta^2}, \quad (9)$$

where  $\mu_{N\Delta}$  is the transition magnetic moment. Depending on the value used for  $\mu_{N\Delta}$ , Eq. (9) gives between<sup>5</sup> +2.4 and<sup>1</sup> +4.0.

The backward spin polarizability  $\gamma_\pi = 4X - 4X_a + \gamma_{M1}^{(\Delta)} + \gamma_\pi^{(\pi\Delta)}$  is clearly dominated by the  $\pi^0$  exchange which gives  $\gamma_\pi^{(\pi^0)} = -4X_a \simeq -45$  for the proton, whereas other terms give  $\gamma_\pi^{(\pi N)} + \gamma_\pi^{(\Delta)} + \gamma_\pi^{(\pi\Delta)} \simeq +7$  to  $+9$ , see Table 1.

### 3 Dispersion relations

A physical origin of the spin polarizabilities which emerges from dispersion relations is very close to that of the HBChPT approach. Generally, nucleon Compton scattering is described by six invariant amplitudes  $A_i(\omega, \theta)$  which are functions of  $\omega$  and the scattering angle  $\theta$ . In the limit of  $\omega \rightarrow 0$ , the non-Born parts  $a_i$  of the amplitudes  $A_{2,4,5,6}$  at zero energy determine the spin polarizabilities which are linear combinations of the constants  $a_i$ .<sup>1</sup> Three of these amplitudes ( $A_{4,5,6}$ ) satisfy unsubtracted dispersion relations at fixed  $\theta = 0$  which give

$$a_i = \frac{2}{\pi} \int_{\text{thr}}^{\infty} \text{Im } A_i(\omega, 0^\circ) \frac{d\omega}{\omega}, \quad i = 4, 5, 6, \quad (10)$$

and allow to determine through unitarity and photoproduction amplitudes those combinations of spin polarizabilities which do not depend on  $a_2$ . They are

$$\gamma_{E1} + \gamma_{M1}, \quad \gamma_{E2} + \gamma_{M2}, \quad \gamma_{E1} + \gamma_{E2} \quad (11)$$

(and thus also  $\gamma_{M1} + \gamma_{M2}$  and the forward spin polarizability  $\gamma$ , Eq. (4)). A determination of the last parameter  $a_2$  which enters the polarizability  $\gamma_\pi$ , Eq. (5), is not reduced to the knowledge of photoproduction only and needs

also a knowledge of  $t$ -channel exchanges, including  $\pi^0$ ,  $\eta$ ,  $\eta'$ . A dispersion relation at the backward angle  $\theta = 180^\circ$ , which includes both  $s$ - and  $t$ -channel parts,

$$\gamma_\pi = \int_{\text{thr}}^\infty \sqrt{1 + \frac{2\omega}{M}} \left(1 + \frac{\omega}{M}\right) \sum_n P_n \left( \sigma_{3/2}^n(\omega) - \sigma_{1/2}^n(\omega) \right) \frac{d\omega}{4\pi^2 \omega^3} + \gamma_\pi^t, \quad (12)$$

has recently been shown to provide a reliable way to calculate  $a_2$  and  $\gamma_\pi$  using the photoabsorption cross sections with the total helicity 1/2 and 3/2 and with the relative parity  $P_n$  of the produced states  $n$ .<sup>6</sup>

In the chiral limit, the unitarity gives the imaginary parts of the amplitudes  $A_i$  to order  $\mathcal{O}(p^3)$  as a phase space  $\mathcal{O}(p)$  times a square of the pion photoproduction amplitude  $\mathcal{O}(p)$ , the latter being given by tree diagrams of  $\gamma N \rightarrow \pi N$  calculated with the static nucleon. Accordingly, the unsubtracted fixed- $t$  dispersion relations (10) give in the chiral limit

$$\begin{aligned} \gamma_i &= C \int_0^1 G_i(v) v^2 dv, \quad C = \frac{e^2 g_A^2}{16\pi^3 m_\pi^2 f_\pi^2}, \quad G_{E1}(v) = -\frac{1}{2} - \frac{1-v^2}{4v} \ln \frac{1+v}{1-v}, \\ G_{E2}(v) &= G_{E1}(v) + 1, \quad G_{M1}(v) = G_{M2}(v) = 0, \end{aligned} \quad (13)$$

where  $v$  is the pion velocity (cf. <sup>8</sup>). Eq. (13) exactly reproduces the loop contributions (7) of HBChPT to the “good” polarizabilities (11). For  $\gamma_\pi$ , however, Eq. (13) does not work because the amplitude  $A_2$ , which gets also a contribution from the  $\pi^0$  exchange, does not obey Eq. (10). In this case the backward dispersion relation (12) can be used instead very efficiently. It gives in the chiral limit:

$$\gamma_\pi = C \int_0^1 G_\pi(v) v^2 dv + \gamma_\pi^t, \quad G_\pi(v) = \frac{1-v^2}{2v} \ln \frac{1+v}{1-v}, \quad (14)$$

where  $\gamma_\pi^t = -4X_a$  is the contribution of the  $t$ -channel  $\pi^0$ -exchange found with the WZW coupling, and again exactly matches Eq. (7).

With more realistic photoproduction amplitudes like those from the SAID code,<sup>9</sup> and with additional  $\eta$  and  $\eta'$  exchanges, the dispersion relations predict large deviations from Eq. (7).<sup>1,3,6</sup> A major part of these deviations is caused, however, by the  $\Delta$  resonance, so that the obtained results for  $\gamma$ 's turn out to be not too far from predictions of HBChPT with the  $\Delta$ -isobar explicitly included through the  $\epsilon$ -expansion, see Table 1.

#### 4 Higher resonances in NQM

For HBChPT to have a practical success, contributions of resonances, which are normally treated as low-energy constants (or counter-terms), have to be

small. Therefore, the important question is what are contributions of higher resonances (beyond  $\Delta(1232)$ ) to the spin polarizabilities. A non-relativistic quark model (NQM) can be used to answer this question.

Considering the Hamiltonian of the NQM and doing a special gauge transformation which makes the Born contribution manifestly separated,<sup>10</sup> one can find the low-energy expansion of the Compton scattering amplitude and obtain explicit formulas for all the spin polarizabilities.<sup>11</sup> They involve dipole and quadrupole matrix elements of the electromagnetic current. The  $M1 \rightarrow M1$  and  $E2 \rightarrow M1$  transitions are saturated by the  $\Delta$ -isobar excitation and are given by the transition magnetic dipole  $\mu_{N\Delta}$  and the electric quadrupole  $Q_{N\Delta}$  moments. The rest contributions can be evaluated using a closure over the  $1\hbar\omega$  shell which leads to many simplifications. Then the final result reads

$$\gamma_{M1} = \frac{\mu_{N\Delta}^2}{4\pi\Delta^2}, \quad \gamma_{E2} = -\frac{\mu_{N\Delta}Q_{N\Delta}}{8\pi\Delta}, \quad \gamma_{E1} = -\gamma_{M2} = -\frac{e^2(1+\tau_3)}{72\pi m_q^2\omega_q^2}, \quad (15)$$

where  $m_q \simeq 340$  MeV is the quark mass and  $\omega_q \simeq 500$  MeV is the oscillator frequency. The term with  $1/\omega_q^2$  represents a joint effect of a few nucleon resonances in the mass range between 1520 and 1700 MeV and is absent for the neutron ( $\tau_3 = -1$ ).

In the present context, the only important thing is that all the resonances lying beyond the  $\Delta$  region have a small effect on the spin polarizabilities, see Table 1. This finding supports the physical conclusion inferred from dispersion calculations that the values of the spin polarizabilities of the nucleon are mostly related with low-energy excitations and long-range periphery of the nucleon (i.e. low-energy  $\pi N$  production and the  $\pi^0$ -exchange). This explains why HBChPT to leading non-vanishing order gives already the results which are rather close to those found through a much more detailed dynamical input.

## 5 Conclusions

The presented arguments, both original and borrowed from other authors, show that the spin polarizabilities  $\gamma_i$  mostly depend on the low-energy dynamics of the nucleon. Different evaluations of the spin polarizabilities agree with each other. For the backward spin polarizability  $\gamma_\pi$ , they suggest a much larger value than that found in the recent LEGS experiment.<sup>4</sup> In view of the large discrepancy between the fundamental predictions for  $\gamma_\pi$  done on the basis of HBChPT and dispersion relations, it would be important to test the experimental number (6) in a dedicated experiment with polarized particles.

## Acknowledgments

I am pleased to thank organizers of the Conference for their invitation and support. Useful discussions with A.M. Nathan and G. Krein are very appreciated. This work was partially supported by the RFBR grant 98-02-16534.

Table 1: Spin polarizabilities of the proton in HBChPT and in the dispersion theory. The columns  $(\pi N)$ ,  $(\Delta)$ ,  $(\pi\Delta)$  and  $(\pi^0)$  give the HBChPT contributions<sup>5</sup> to order  $\mathcal{O}(p^3)$  (for  $\Delta$ , a “large”  $M1$  coupling was used and the  $E2$  excitation was also included, see<sup>1</sup>);  $N^*$  is an NQM estimate for higher resonances;  $DR(s)$  are contributions of photoproduction in the dispersion theory found with the SAID multipoles;  $DR(t)$  is the  $t$ -channel contribution for the backward dispersion relation (includes  $\pi^0$ ,  $\eta$ ,  $\eta'$ ). For all but the last line ( $\gamma_\pi$ ), dispersion relations used are those at fixed  $\theta = 0$ , Eq. (10); for  $\gamma_\pi$ , it is Eq. (12).

$10^{-4} \text{ fm}^4$	$(\pi N)$	$(\Delta)$	$(\pi\Delta)$	$N^*$	$(\pi^0)$	$DR(s)$	$DR(t)$
$\gamma_{E1} + \gamma_{M1}$	-6.7	4.0	0.45	-0.4		-0.7	
$\gamma_{E2} + \gamma_{M2}$	2.2	0.75	-0.23	0.4		2.2	
$\gamma_{E1} + \gamma_{E2}$	-4.4	0.75	0.21	-0.4		-1.5	
$\gamma_{M1} + \gamma_{M2}$	0	4.0	0	0.4		3.0	
$\gamma$	4.4	-4.75	-0.21	0		-1.5	
$\gamma_\pi$	4.4	4.75	-0.21	0	-45.3	7.0	-46.6

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